A spam filter is designed by looking at commonly occurring phrases in spam. Suppose that 80% of email is spam. In 10% of the spam emails, the phrase “free money” is used, whereas this phrase is only used in 1% of non-spam emails. A new email has just arrived, which does mention “free money”. What is the probability that it is spam?

Let's create some notations

S - an email is a spam FM - 'free money' phrase is used N - an email is normal (not a spam)

We are asked to find 𝑃(𝑆|𝐹𝑀) Notice 𝑃(𝑆)+𝑃(𝑁)=1 and apply Baysian formula

𝑃(𝑆|𝐹𝑀)=𝑃(𝐹𝑀|𝑆)/𝑃(𝑆)𝑃(𝐹𝑀|𝑆)𝑃(𝑆)+𝑃(𝐹𝑀|𝑁)𝑃(𝑁) where

𝑃(𝐹𝑀|𝑆)=10%, 𝑃(𝐹𝑀|𝑁)=1%

Plug all back to the conditional probability, I got

𝑃(𝑆|𝐹𝑀)=40/41